Order at Arbitrarily High Temperatures

Zohar Komargodski

Simons Center for Geometry and Physics

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This presentation is based on

★ Phys.Rev.D 102 (2020), arXiv: 2005.03676 with **Noam Chai, Soumyadeep Chaudhuri, Chang-Ha Choi , Eliezer Rabinovici, and Misha Smolkin.**

★ Phys.Rev.Lett. 135 (2025), arXiv: 2412.09459 with **Fedor Popov.**

★ Nature Commun, arXiv: 2503.22789 **Yiqiu Han, Xiaoyang Huang, Andrew Lucas, Fedor Popov.**

★ Work to appear with **Xiaoyang Huang, Andrew Lucas, Fedor Popov, Tin Sulejmanpasic.**

- General principles in thermal physics and the no-hair theorem
- "Arithmetic Ising Model"
- Bi-conical magnets
- Other high-temperature orders.

Many systems exhibit symmetry breaking (order) at zero temperature. For instance, ferromagnets, massless QCD, anti-ferromagnets, superfluids etc.
At zero temperature we can also encounter topological order.

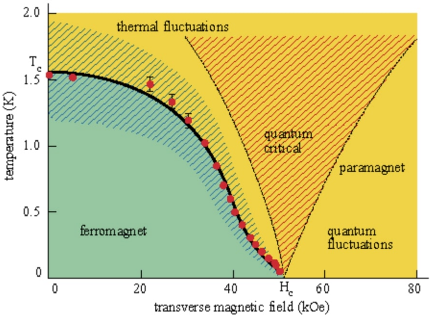
We usually think that if we heat these systems up, i.e. study instead of the vacuum the thermal state

$$\rho = \frac{1}{Z} e^{-\beta H}$$

then all the symmetries are restored for sufficiently small β. We also expect topological order to disappear at sufficiently small β.

Indeed, most phase diagrams for quantum critical points look like this (phase diagram of $LiHoF_4$ as measured by Bitko et al)

Figure 1: **Quantum criticality in a ferromagnet.**



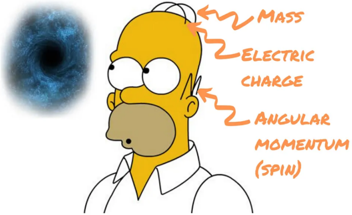
Why should symmetry be restored at high enough temperature?

At finite temperature we minimize

$$F = E - ST \ .$$

At large T the dominant contribution is from high entropy states and those are disordered. We are usually taught that by definition entropy is higher in disordered states.

For holographic systems, high temperature states are dual to a (large) black hole in AdS. For the latter, it is widely believed that there is a no hair theorem and hence no symmetry breaking.



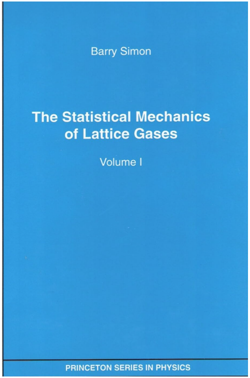
More precisely, UV complete stringy models do not have hairy large black holes (at least so far), but one can write down bottom up effective supergravity theories with hairy large black holes [Buchel].

Under some assumptions there are rigorous theorems that

$$\rho = \frac{1}{Z} e^{-\beta H}$$

is trivial for sufficiently small β.

Sometimes we refer to such theorems as the "uniqueness of the Gibbs state." Or "Dobrushin theorem." A quantum version is the "heat death of entanglement" [Martin Kliesch, Christian Gogolin, Michael Kastoryano, Arnau Riera, and Jens Eisert].



In the classical version one proves that sites are uncorrelated at high temperature

$$\beta < \beta_c, \ |x-y| \gg \beta : \quad \langle U(x)V(y)\rangle_\beta \to \langle U(x)\rangle_\beta \langle V(y)\rangle_\beta$$

for any operators U, V

In the quantum version one further proves that entanglement cannot survive heat

$$\beta < \beta_c \ , \quad \frac{e^{-\beta H}}{Tr(e^{-\beta H})} = \sum_i p_i \rho_A^i \otimes \rho_B^i \ .$$

This means that the density matrix is separable.

The theorems about the uniqueness of the Gibbs state at $\beta < \beta_c$ have important assumptions:

1

The uniform distribution $\mathbb{I}$ over the space of states Ω exists.

2

$\Omega = \times\Omega_i$, i.e. there are no constraints and the space of states is a direct product.

In summary: experiments, the no-hair theorem, thermodynamic arguments, and in some cases rigorous theorems all suggest that the expectation values of order parameters must vanish at sufficiently high temperatures.

In quantum field theory the assumptions of the Dobrushin-type theorems are always violated. They can be also violated on the lattice.

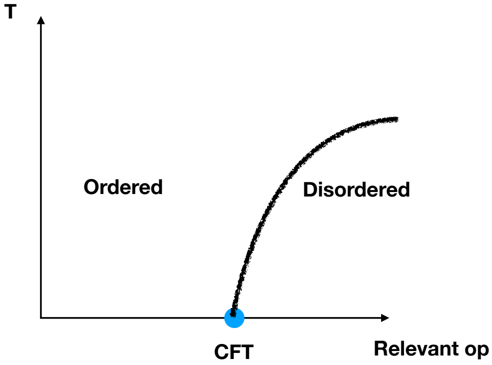
We will therefore pose three questions

Does large entropy really mean lack of order?

Are there local lattice models which are ordered at high temperature?

Are there nontrivial CFTs which break a global symmetry at finite temperature?

The phase diagram that could emerge, if we find counterexamples:



Let us first consider classical stat-mech in 2d

$$\mathbb{P}(\mathbf{n}) = \frac{\mathrm{e}^{-\beta H(\mathbf{n})}}{Z(\beta)} \tag{1}$$

where $H(\mathbf{n})$ is the energy (Hamiltonian) of the microstate, and the partition function

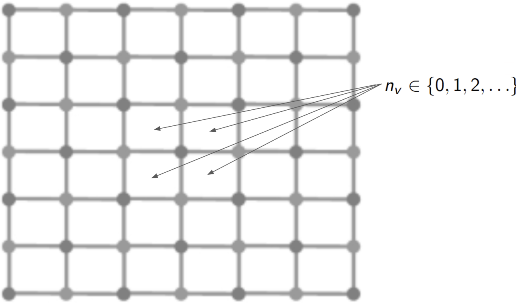
$$Z(\beta) = \sum_{\mathbf{n}} \mathrm{e}^{-\beta H(\mathbf{n})} = \mathrm{e}^{-\beta F(\beta)} \tag{2}$$

both normalizes the probability distribution, and defines the free energy F.

On the $L \times L$ square lattice, at each plaquette we can have $n_v \in \{0, 1, 2, \ldots\}$ bosons. We choose

$$H = U \sum_{u \sim v} n_u n_v + \sum_v n_v. \tag{3}$$

This means that bosons repel on adjacent plaquettes and there is an overall chemical potential.



$$H = U \sum_{u \sim v} n_u n_v + \sum_v n_v . \qquad (4)$$

Another possible interpretation is that n_v is an internal level of an atom sitting at the plaquette v.

We call this model the "Arithmetic Ising Model" (AIM).
Consider the phase with uniform density $\langle n_u \rangle \equiv \bar{n}$. In mean field theory we find at very high temperature $\bar{n} = \sqrt{\frac{T}{2U}}$.
Furthermore, the partition function at high temperature is given by

$$\log Z_{uniform} = \frac{1}{2} L^2 \log\left(\frac{T}{2U}\right) \ . \tag{5}$$

Now let us consider a checkerboard phase. Assume that $\langle n_x \rangle \sim \bar{n}_A$ whenever $x \in A$, where A is one sub-lattice, while $\langle n_x \rangle = 0$ when $x \in A^c$. We find $\bar{n}_A = T$ and the partition function is given by

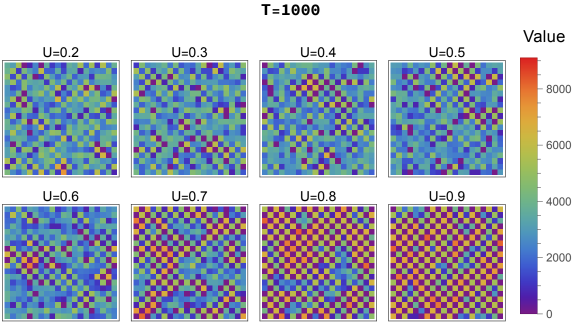
$$\log Z_{solid} = \frac{1}{2} L^2 \log T \ . \tag{6}$$

Hence we have that $Z_{uniform} \geq Z_{gas}$ whenever $U > 1/2$, and the ordered phase is favored at high temperature.

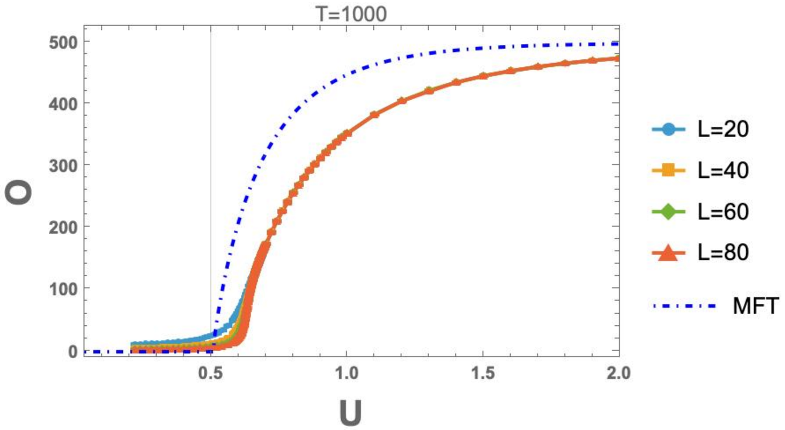
An interesting variant of the model is a "colored AIM" model with k colors. In this colored variant we let $n_x = \sum_{A=1}^{k} n_x^A$, where n_x^A a nonnegative integer. In addition we rescale $U \to U/k$.

The large k limit is solvable and coincides with mean field theory.

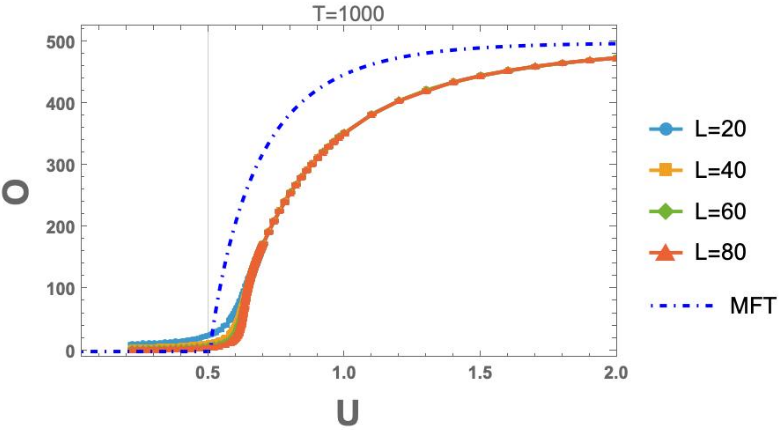
We calculated $1/\mathrm{k}$ corrections and verified that MFT predictions only slightly change.



Result of Monte-Carlo. We see solidification at high temperature!



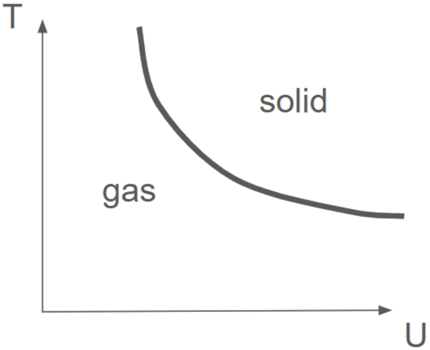
While MFT gives $U_c = 1/2$, MC shows $U_c \sim 0.6$, and a second order transition at any temperature. O is the order parameter for one-site translational symmetry breaking.



Interestingly as we increase the temperature the mean field result $U_c = 1/2$ is approached. Maybe MFT is exact at extremely high temperatures here?!

Summary phase diagram:

$$H = U \sum_{u \sim v} n_u n_v + \sum_v n_v$$



The mechanism is that order in one degree of freedom can enable other degrees of freedom to strongly fluctuate. “Sacrificing” some subset of the degrees of freedom can be therefore beneficial.

That is how we got a solid that would not melt.

The same ideas can be used to engineer systems where the typical high energy states are ferromagnetic, superconducting...

Consider the Landau-Ginzburg theory of the $O(N)$ order parameter at criticality

$$S_{O(N)} = \int d^2x dt \left(\frac{1}{2}(\partial\vec{\phi})^2 + (\vec{\phi}^2)^2 \right) \ .$$

This model by itself is in a trivial phase at high temperature, with no entanglement or symmetry breaking. Indeed it has a thermal mass [Sachdev...]

$$m_T = 2\log\left(\frac{1+\sqrt{5}}{2}\right) T + \cdots \ ,$$

$\langle\phi(x)\phi(0)\rangle_T \sim e^{-m_T|x|}$.

We will slightly modify the model and see that it develops very unusual thermal properties.

Our modification is to add a real field Ψ to the $O(N)$ model as

$$S_{O(N)} + \int d^2xdt \ \frac{1}{2}(\partial\Psi)^2 - \underbrace{\lambda \int d^2xdt\vec{\phi}^2\Psi^2} \ .$$

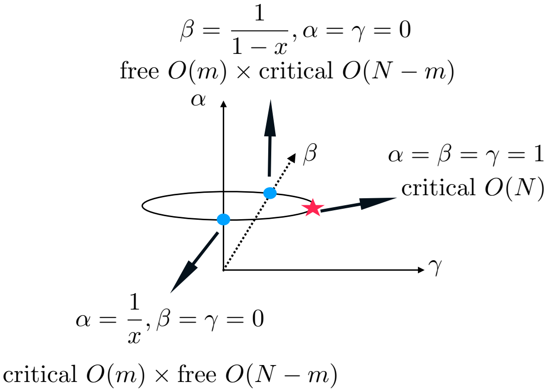
From the large N solution we know

$$\Delta(\vec{\phi}^2) = 2 - \frac{32}{3\pi^2 N} + \cdots \ .$$

The interaction with Ψ is relevant for all N.

$$S_{O(N)} + \int d^2xdt\ \frac{1}{2}(\partial\Psi)^2 - \underbrace{\lambda \int d^2xdt\vec{\phi}^2\Psi^2}\ .$$

The dynamics can be analyzed in the ϵ expansion in $2.99+1$ space-time dimensions or in the large N expansion directly in 2+1 d.



In the large N expansion in 3d

$$\beta_\lambda = \frac{32}{3\pi^2 N}(\lambda - \lambda^3) + \cdots$$

The most interesting fixed point for us is that with

$$\lambda = 1 + ...$$

This is a bi-conical fixed point with symmetry

$$O(N) \times \mathbb{Z}_2 \ .$$

Magnets with such symmetry are known for many years.

To fully establish the fixed point, the sextic interaction

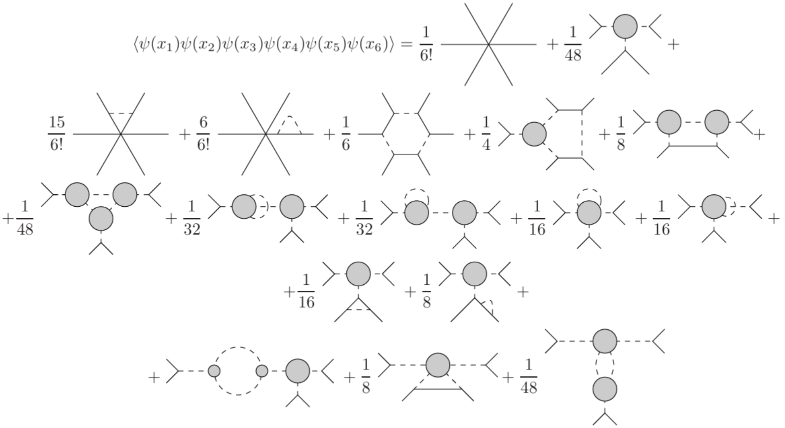
$$g \int d^2x dt \ \Psi^6$$

has to be determined as well. This is quite cumbersome*, but there is a fixed point with

$$g_* = \frac{13560}{6! N^2} \ .$$

We see that the potential is bounded from below and the theory is healthy.

$$\langle \psi(x_1)\psi(x_2)\psi(x_3)\psi(x_4)\psi(x_5)\psi(x_6)\rangle =$$



(A2)

(A3)

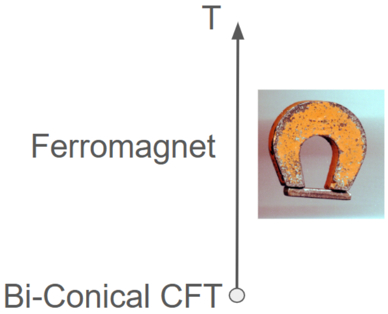
(A4)

(A5)

We can determine the finite temperature behavior by solving the gap equations (Schwinger-Dyson) equations. We find

$$\langle \Psi \rangle \sim 0.8\, T^{1/2} \ .$$

The theory is gapless critical at $T = 0$ and ferromagnetic at any temperature $T \neq 0$!



What are the values of N for which this remarkable behavior occurs?

Indications in [Bilal Hawashin, Junchen Rong, and Michael M. Scherer] that it is valid for $N > 10$.

Some calculations we did suggest $N > 4$.

The continuum theory is an approximation. The lack of symmetry restoration in the continuum means that the transition temperature in these materials might be anomalously high.

Finally, let us do something similar for superconductivity. The idea is the same: make it favorable for the system to form Cooper pairs even at high temperature because that would open up some additional fluctuations.

We consider the BCS order parameter

$$\Delta = \frac{1}{N} \sum_{a=1}^{N} \psi_{\uparrow}^{a} \psi_{\downarrow}^{a} \ .$$

Usually we have superconductivity with $\Delta \neq 0$ for

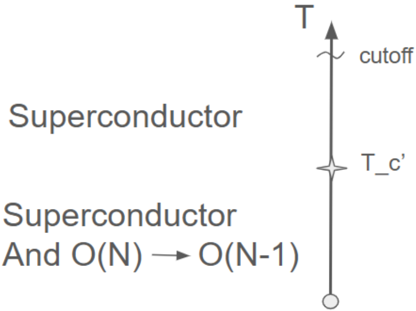
$$T < T_c = \omega_* e^{-\frac{1}{g\nu}}$$

(g the attractive interaction and ν the density of states).

Now assume that the order parameter talks to some internal $O(N)$ degrees of freedom through

$$-\lambda \int d^3xdt\vec{\phi}^2|\Delta|^2 \ .$$

This model has an interesting phase diagram:



We have provided a general mechanism that allows for typical excited states to be ordered. We have discussed spatial order (solid), magnetic order (ferromagnet), and Cooper pairs (superconductor). Some obvious open questions

- Can any of this be realized in real life?
- Is the transition in the $n + nn$ model everywhere second order?
- What about holographic theories? UV complete gauge theories in 3+1 dimensions? [See Choi et al, and also Bajc, Lugo, Sannino]
- What is so special about the Bi-Conical model? is this behavior actually generic?

Thank You!